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Measuring external stability in one-to-one matching

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Abstract

We consider a one-to-one matching model where the population expands with the arrival of a new individual. Individuals in this population are matched according to the deferred acceptance (DA) algorithm where men propose and women accept or reject. Using computer simulations of this model, we study how the matching utilities of individuals are affected when the initial size of the population or the degree of heterogeneity in preferences change. We also investigate to what extent the preference heterogeneity affects the difference between the likelihoods of matching obtained under the DA algorithm and the Top Trading Cycle algorithm.

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1. Introduction

It is well known that for every two-sided (heterosexual) population in which individuals' preferences satisfy some mild assumptions there always exists a one-to-one (monogamic) matching that is stable. Under such a matching no pair of individuals would prefer each other to their mates and no individual would prefer staying single to his/her match. This celebrated result is due to Gale and Shapley (1962), who provide the proof by showing that a dynamic procedure —called the deferred acceptance (DA) algorithm— always produces, after some finite iterations, a stable matching. According to this algorithm, individuals in one side of the population propose for matching and individuals in the other side reject or (tentatively or permanently) accept these proposals. Because of its simplicity and computational power, this algorithm has been very popular and useful —since it was proposed (and in some markets even before that)— to solve many theoretical and practical matching problems (e.g. Roth 1984, 2002, 2008; Balinski and Sönmez 1999; Abdulkadiroğlu, Pathak, and Roth 2005; and Pathak and Sönmez 2008).

In this paper, we investigate —with the help of computer simulations— how the stable matching generated by the DA algorithm is affected with the arrival of a new individual (either a man or a woman) at the population. We assume that the original population involves equal number of men and women to be able to identify the asymmetric effects of the arrival of the new individual on the average matching utilities of men and women. Ashlagi et al. (2017) show that even the smallest asymmetry in the population with respect to the number of men and women leads to a small core even when preferences are heterogenous, also leading to a huge difference in the average matching utilities of men and women at the core matchings. In our paper, we are only interested in the stable (core) matching produced by the DA algorithm (with men or women proposing) and how the arrival of a man or a woman affects the percentages of men and women who are made better off according to this algorithm. However, we should note that when the population expands, there are two ways to obtain a new stable outcome. The first one is to use the stable matching outcome of the DA algorithm (with men or women proposing) for the original population as a benchmark, and to run the DA algorithm as in Blum et al. (1997) for the expanded population. The second way is to use as benchmark the empty matching where all men and women are available, and to run the DA algorithm for the expanded population.¹ In our paper we use the second way, which is applicable to situations where matching contracts involve a fixed time component or individuals to be matched with have no commitments; i.e., they can unilaterally end the matching relationship at any time without any cost.

Using our model, we study the two-sided welfare effects of population expansion. A theoretical work of Crawford (1991) shows that in a many-to-one matching market

¹Clearly, these two ways would have the same outcome when individuals' preferences are homogenous. Future research may study how apart their outcomes would be when preferences are heterogenous.

involving a finite number of firms and workers, if all preferences are strict, then adding one or more worker (firm) to the market makes the firm-optimal stable outcome weakly worse (better) for all workers and weakly better (worse) for all firms. Simulating situations where the matching population expands by the arrival of a man (or a woman), we investigate how the theoretical results of Crawford (1991) are affected under weak preferences when these preferences can also involve any degree of heterogeneity.

Using our model, we also measure how heterogeneity in preferences affects the matching performance of the DA algorithm in comparison to that of the Top Trading Cycle (TTC) algorithm, another well-known matching procedure in the literature.² The TTC algorithm, which was attributed by Shapley and Scarf (1974) to David Gale, was initially formulated to find a core allocation in one sided markets (e.g. a housing market) where each individual owns an indivisible object (a house).³ When preferences are strict, this algorithm yields the unique allocation in the core (Roth and Postlewaite 1977) and also becomes strategyproof (Roth 1982); i.e., under this mechanism it becomes a dominant strategy for every individual to state his/her true preferences. Moreover, it is the only mechanism that is Pareto efficient, individually rational, and strategy proof (Ma 1994).

For one-to-one matching in marriage markets, we can obtain a TTC algorithm if we modify the algorithm proposed by Abdulkadiroglu and Sönmez (2003) for school choice, by setting the capacity of each school to one. The algorithm of Abdulkadiroglu and Sönmez (2003) allows students to trade their priorities at schools. For marriage markets, we can define the priorities of individuals using their preferences, unless they are given to us exogenously. That is, given any pair of individuals i and j of opposite genders, we can set the priority of i at j to the rank of i from top in the preference relation of j . Then, we can formulate two versions of TTC algorithm, namely TTC-M and TTC-W, depending upon whether men (M) or women (W) are allowed to trade their priorities. Given the priorities of men at women, the TTC-M Algorithm can be described as follows. Initially, no individual is matched. Each unmatched man points to his favorite acceptable woman according to his preference relation. Each woman points in her list of acceptable man to the man with highest priority. (Individuals whose lists of acceptable partners are empty point to themselves.) There is at least one cycle. Allowing the men in a cycle to trade their priorities (according to the preference relations of women), every individual in a cycle is matched to someone (possibly himself/herself) and is removed from the society of unmatched individuals. The algorithm is iterated with the next set of cycles until there remains no cycle. Clearly, one can obtain the TTC-W algorithm from the TTC-M algorithm by interchanging the roles of men and women, i.e., allowing the women in a cycle to trade their priorities

²This comparison, as well as the addition of a single individual to the matching population, were suggested by an anonymous reviewer of this article.

³In a housing market, an allocation is in the core if no coalition of individuals can improve upon it by swapping their own houses.

(according to the preference relations of men). When the preferences are homogenous, the DA-MP and TTC-M algorithms (likewise the DA-WP and TTC-W algorithms) always result in the same stable matching. Through simulations, we aim to show how far apart the outcomes of the two algorithms can be when the preferences are partially or completely heterogenous.

While our research question and our findings are novel to the best of our knowledge, the destabilizing effects caused by a change in the matching population are already known in the matching literature since the works of Blum et al. (1997) and Cantala (2004), dealing with one-to-one markets and many-to-one markets respectively. In this literature, the closest work to ours is Gabszewicz et al. (2012), who show by example that entry to the matching population can heavily destabilize one-to-one marriages and suggest a new stability concept, called k -external stability, to account for the disruptive effects of a change in the matching population. In more detail, Gabszewicz et al. (2012) say that the matching population (the marriage market) is k -externally stable if at any stable matching at least k of the matches are not disrupted whenever the matching population expands with the entry of a man and a woman. Using this definition, they show that a matching population with n men and n women would become 0-externally stable (the worst possible case) if all individuals had common (homogenous) preferences over potential mates whereas the degree of external stability would be $n - 2$ (the best possible case) if all individuals had peak load preferences, i.e., a structure of preferences requiring that on a common ranking of individuals the further distant a potential mate is from the rank of an individual, the least preferred it is. We should note that in our study we do not consider all possible stable matchings associated with the matching population; we focus on the stable matching generated by the DA algorithm, only. Hence, when we expand the matching population, we measure the extent of the survival of the matches under this particular matching only, instead of calculating the degree of external stability of the whole matching population. In addition, unlike in Gabszewicz et al. (2012), we consider heterogenous preferences, using a setup we borrow from Saglam (2020, 2019). This setup allows us to vary, in our simulations, the degree of correlation in the preferences from zero to one and to measure the possible effects of this variation on several variables of interest.⁴ Moreover, we measure these effects by varying the initial size of the matching population, as well.

The rest of our paper is organized as follows. Section 2 introduces our model and Section 3 presents our results. Finally, Section 4 concludes.

⁴Using this setup, Saglam (2019) extends the Todd and Miller's (1999) results in mutual sequential mate search obtained under homogenous preferences to the case of heterogenous preferences whereas Saglam (2020) studies how the heterogeneity in individuals' preferences and the intensity of their learning about their own aspirations (before a matching takes place) can affect the likelihood of marriage and divorce as well as the balancedness and the speed of matching under the stable outcome of the DA algorithm.

2. Model

Consider a population involving a finite set of men M and a finite set of women W , where each individual can be matched to another individual only if they are from opposite sexes. Let $\mathcal{N} = M \cup W$ denote this population. Individual $i \in \mathcal{N}$ derives the utility $U_i(j)$ when he/she is matched with individual j , and derives the utility $U_i(i)$ when he/she is unmatched to any individual (and remains single). Given any $X, Y \in \{M, W\}$ with $X \neq Y$, any $i \in X$, and any $j, k \in Y \cup \{i\}$, we say that i prefers j to k if $U_i(j) > U_i(k)$. We assume that each individual in the population knows all relevant utilities for himself/herself before a matching takes place, and these utilities do not change during the matching process.

A matching is a one-to-one function $\mu : \mathcal{N} \rightarrow \mathcal{N}$ such that for each $m \in M$ and $w \in W$, $\mu(w) = m$ if and only if $\mu(m) = w$. In addition, $\mu(m) \notin W$ if and only if $\mu(m) = m$, and $\mu(w) \notin M$ if and only if $\mu(w) = w$. Individuals m and w are matched to each other if $\mu(m) = w$, and individual i is single if $\mu(i) = i$.

A matching μ is said to be *acceptable* for individual i if $U_i(\mu(i)) \geq U_i(i)$; i.e., the utility of individual i from the match $\mu(i)$ is not below the utility from being single. Also, given any matching μ , a man m and a woman w are together called a *blocking pair* for μ if $\mu(m) \neq w$ and $U_i(j) > U_i(\mu(i))$ for $i, j \in \{m, w\}$ with $i \neq j$; i.e., m and w are not matched under μ and they prefer each other to their matches at μ . Given these definitions, a matching μ is said to be *stable* if μ is acceptable for each individual and there exists no blocking pair for μ .

A celebrated result in matching theory, due to Gale and Shapley (1962), shows that there exists a stable matching for every matching population provided that individuals' preferences satisfy completeness and transitivity.⁵ Moreover, under these assumptions, a stable matching can always be obtained as the outcome of a procedure, called the deferred acceptance (DA) algorithm, which has two versions depending upon the roles of men and women. In one of these algorithms, men propose to women and women give rejections or (tentative or permanent) acceptances, while in the other algorithm proposals are given by women and rejections/acceptances by men. The matching outcomes of these two algorithms, even though they are both stable, have different welfare implications. Gale and Shapley (1962) show that if all individuals have strict preferences, the DA algorithm, when men (women) propose, always produces the men-optimal (women-optimal) stable matching; i.e. a stable matching which is preferred by all men (women) to any other stable matching.

⁵Given two alternatives x and y , an individual is said to weakly prefer x to y if he/she strictly prefers x to y or is indifferent between them. The preference ordering of this individual over the set of possible alternatives satisfies *completeness* if for any two alternatives in this set it is true that he/she weakly prefers one of them to the other. Also, his/her preference ordering satisfies *transitivity* if for any three alternatives x, y, z it is true that he/she weakly prefers x to z whenever he/she weakly prefers x to y and weakly prefers y to z .

We will denote by DA-MP (DA-WP) the deferred acceptance with men proposing (women proposing). Below, we describe the DA-MP algorithm with $k \geq 1$ steps. Initially, all individuals are single in this algorithm. (Interchanging the roles of men and women, one can simply obtain the DA-WP algorithm from below.)

Step $k \geq 1$: Every man who is in step 1, or who was rejected in step $k - 1$ when k is at least 2, proposes to his most preferred woman in his updated list of acceptable women (if any). A man makes no proposal if his list is empty. Each woman holds the most preferred acceptable proposal she has received until now and rejects all other proposals. Then, each man rejected in this step deletes the woman who rejected him from his list of acceptable women. (Above, if any individual is indifferent between any two potential mates, he/she is allowed to break the tie arbitrarily.) The algorithm terminates when no further proposal is made by any man, and at this step each woman is matched to the man (if any) whose proposal she is holding.

Gale and Shapley (1962) proved that the DA algorithm described above must always yield, after finite steps, a stable matching. Below, we will investigate, through computer simulations, how the outcome of the DA algorithm changes when the matching population expands with the arrival of a single man and a single woman or with the arrival of a single individual (either a man or a woman), and to what extent the investigated change in the matching outcome is affected by the heterogeneity in preferences.

Varying the level of preference heterogeneity, we will also compare the performance of the Gale and Shapley's DA algorithm—in terms of matching likelihood—with that of the Gale's TTC algorithm, which we reformulate for marriage markets by modifying the TTC algorithm in Abdulkadiroğlu and Sönmez (2003) proposed for school choice. We do this by simply setting the capacity of each school in Abdulkadiroğlu and Sönmez (2003) to one and defining the priorities using the preference relations. That is, we assume that given any pair of individuals i and j of opposite genders, the priority of i at j is equal to the rank of i from top in the preference relation of j . Given these modifications, we can consider two versions of the TTC algorithm, namely TTC-M and TTC-W, depending upon whether men (M) or women (W) are allowed to trade their priorities. Below, we describe the TTC-M Algorithm with $k \geq 1$ steps. (The women-proposing version, TTC-W, can be simply obtained by interchanging the roles of men and women.) Initially, all individuals are single in this algorithm.

Step $k \geq 1$: Every man who is in step 1, or who was unmatched in step $k - 1$ when k is at least 2, points to his favorite acceptable woman according to his preference relation. Every woman who is in step 1, or who was unmatched in step $k - 1$ when k is at least 2, points in her list of acceptable man to the man with highest priority. (Individuals whose lists of acceptable partners are empty point to themselves.) There is at least one cycle. Allowing the men in a cycle to trade their priorities (according to the preference relations of women), every individual in a cycle is matched to someone

(possibly himself/herself) and is removed from the society of unmatched individuals and from their lists of acceptable partners. (If any individual is indifferent between any two potential mates, he/she is allowed to break the tie arbitrarily.) The algorithm is iterated with the next set of cycles until there remains no cycle.

Clearly, the number of steps in the TTC algorithms cannot exceed the maximum of the number of men and the number of women. Also, the DA-MP and TTC-M algorithms (likewise the DA-WP and TTC-W algorithms) lead to the same matching outcome when the preferences of individuals are homogeneous. Our simulations will investigate how far apart their outcomes can be when the preferences are heterogenous.

3. Results

Below, we first study through computer simulations how the external stability of the matching outcome of the DA-MP algorithm is affected by a change in the size of heterogeneity in the preferences of individuals when the initial size of the matching population is also varied. We conduct all simulations using codes written in GAUSS Software Version 3.2.34 and MATLAB Software Version R2019b. To run the TTC-M algorithm for marriage markets, we use a modified version of the MATLAB code written by Zweifel (2009) for the school choice problem. On the other hand, for the DA-MP algorithm we use our own codes in GAUSS. (All program codes and the resulting data are available upon request.)

We assume that the number of men and the number of women in the initial population are the same and denoted by n ; and we vary in our simulations the number n between the integers 2 and 100. We model the preferences of individuals in the initial and extended population using the preference structure in Saglam (2020, 2019). Formally, for any two individuals, say i and j , with opposite genders, we assume that the match utility $U_i(j)$ of individual i derived from individual j satisfies

$$U_i(j) = \omega U^c(j) + (1 - \omega) U_i^p(j),$$

where $U^c(j)$ denotes the common part of the utility any individual in the gender class of i can derive from individual j , $U_i^p(j)$ denotes the private part of the utility only individual i can derive from individual j , and ω denotes a weight parameter that is, for simplicity, assumed to be common for all individuals in the population. In our simulations, the utilities $U^c(j)$ and $U_i^p(j)$ in the above expression are always randomly drawn from a uniform distribution of values in $[0, 100]$. On the other hand, the weight parameter ω is varied inside the set of values $\{0, 1/3, 2/3, 1\}$. It is clear that the higher the value of ω , the less heterogenous the preferences of individuals. In particular, $\omega = 0$ and $\omega = 1$ induce completely heterogenous (perfectly uncorrelated) and completely homogenous (perfectly correlated) preferences, while $\omega = 1/3$ and $\omega = 2/3$ induce partially heterogenous (imperfectly correlated) preferences. For each specification of

n and ω in their domains, we randomly draw 100 samples of utility profiles in which individual utilities are uniformly distributed in their domains.

Now, we will consider the expansion of the population by the arrival of a single individual and study how the welfares of individuals are affected. First, we let the newly arrived individual be a man, and simulate the outcome of the DA-MP algorithm before and after his arrival starting from an empty matching. Crawford's (1991) comparative statics results imply that –when preferences of individuals are strict– the arrival of a new man would make the outcome of the DA-MP algorithm weakly worse for all men –for which the competition over women has become increased– and weakly better for all women. Even though the preferences are not strict in our model, our simulations yield similar results, which we illustrate in Figures 1 and 2. Figure 1 illustrates that the percentage of men who become better (i.e. obtain a higher matching utility) with the arrival of a new man is very low if the population size is small, and exactly zero otherwise. On the other hand, a non-negligible fraction of women becomes strictly better off with the arrival of a new man, more so when the population size is not very high. Furthermore, Figure 2 shows that no woman in the population ever becomes strictly worse off, whereas a non-negligible fraction of men always becomes strictly worse off, with the arrival of a new man. Figures 1 and 2 also show that the effect of heterogeneity in preferences is somewhat visible, but quite low.

In Figures 3 and 4, we consider the welfare effects of the expansion of the matching population by the arrival of a woman. We obtain entirely opposite results in comparison to those in Figures 1 and 2, since the arrival of a new woman reduces the competition among the men, hence their welfares almost always weakly improve. Most of the men preserve their marital status and their wives (if any) before the expansion, while some of them strictly improve their marital position (either by remarrying with a more desirable wife or by changing their status from ‘single’ to ‘married’). In contrast, each woman faces more competition and receives less proposals from men under the DA-MP algorithm, and consequently they become weakly worse off.

Finally, we investigate how far apart the matching likelihoods of the TTC-M and DA-MP algorithms are when the preferences of individuals are heterogenous. Panel (a) of Figure 5 shows that if preferences are completely heterogenous ($\omega = 0$), then the TTC-M algorithm yields a higher matching likelihood, provided that the population size is not too small. In the other extreme case where preferences are completely homogenous ($\omega = 1$), the two algorithms always produce the same number of matchings, as expected (see Figure 5-d). Moreover, the four panels of Figure 5 together show that the superior performance of the TTC-M algorithm becomes more and more visible when the level of heterogeneity becomes higher (i.e., ω becomes lower).

4. Conclusion

In this paper, we have studied how the stable matching outcome generated by the DA algorithm is affected when the matching population expands with the arrival of a single

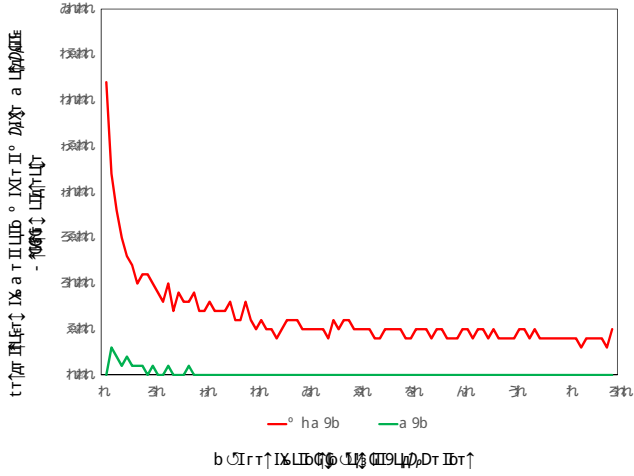
individual (either a man or a woman). As benchmark for the expanded population, we have used the empty matching where all men and women are available. Our Monte Carlo simulations have showed that the theoretical results of Crawford (1991) dealing with –under strict preferences assumption– the welfare effects of population expansion almost always hold in our model that allows weak preferences. In more detail, we have found that the arrival of a new man makes the outcome of the DA-MP algorithm weakly worse for almost all men and weakly better for all women. In line with Crawford (1991), we have also found that if the newly arrived individual is instead a woman, then her arrival makes the outcome of the DA-MP algorithm weakly better for almost all men and weakly worse for almost all women. While these results only show how the arrival of a man or a woman affects the percentages of men and women who are made better off under the DA-MP algorithm, future research may study, following the work of Ashlagi et al. (2017), how the difference of the average utilities of men and women would be affected under the DA-MP or TTC-M algorithms.

In our paper, we have also run simulations to study how far apart the matching likelihoods of the TTC and DA-MP algorithms are when the preferences of individuals are heterogenous. We have found that when preferences are (sufficiently) heterogenous, the matching likelihood of the TTC-M algorithm is always (slightly) above the matching likelihood of the DA-MP algorithm. On the other hand, the matching performances of the two algorithms are (almost) the same when preferences are (sufficiently) homogenous.

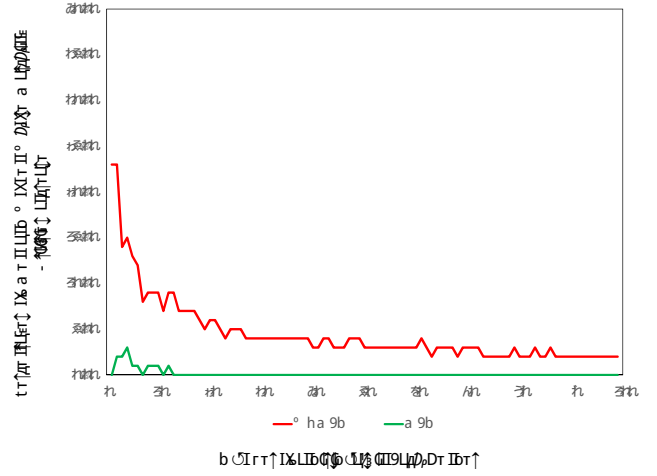
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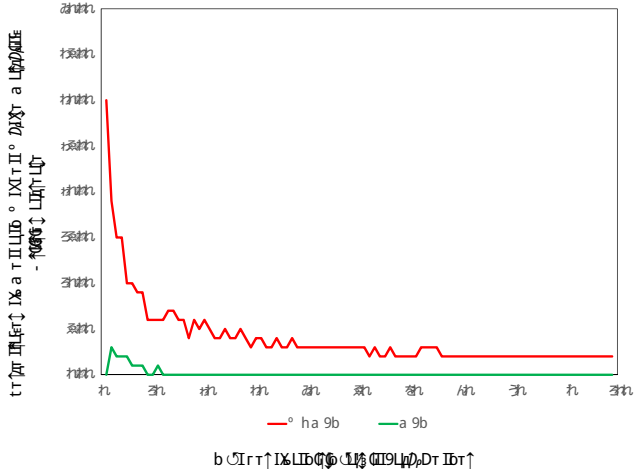
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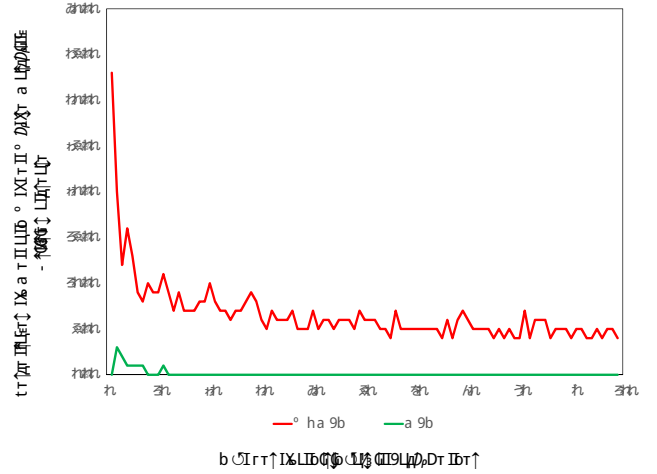
(a) $w = 0$



(b) $w = 1/3$



(c) $w = 2/3$

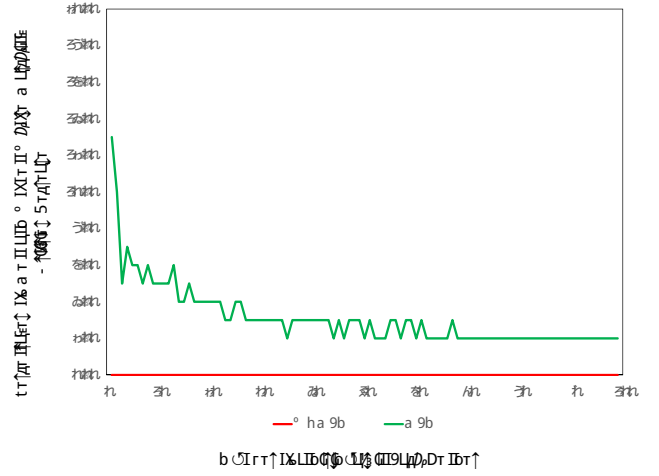


(d) $w = 1$

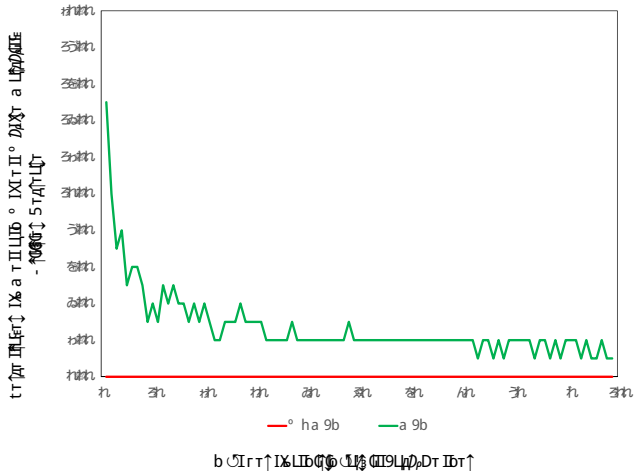
Figure 1. Percentages of Men and Women Who Are Made Better Off by the Outcome of the DA-MP Algorithm After the Arrival of a Man



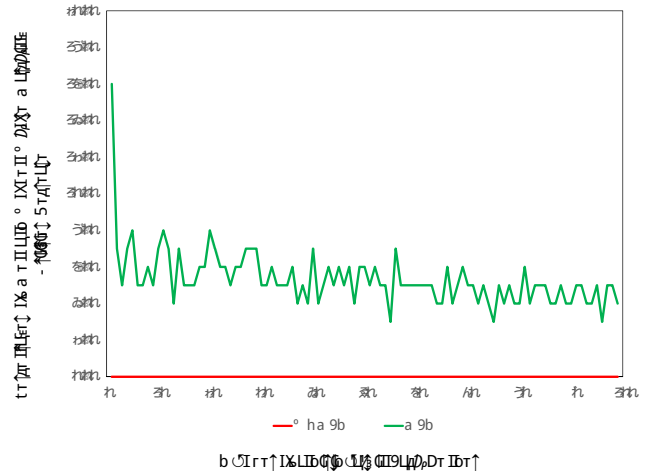
(a) $w = 0$



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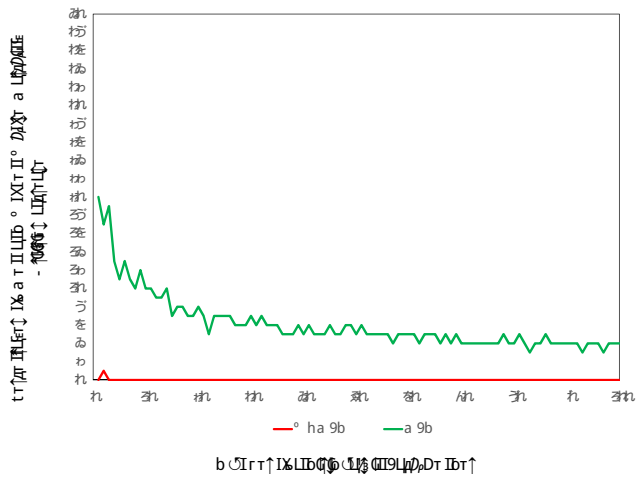


(c) $w = 2/3$

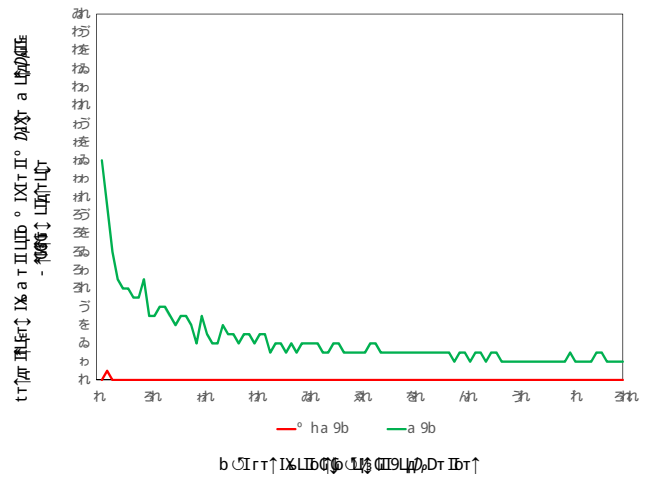


(d) $w = 1$

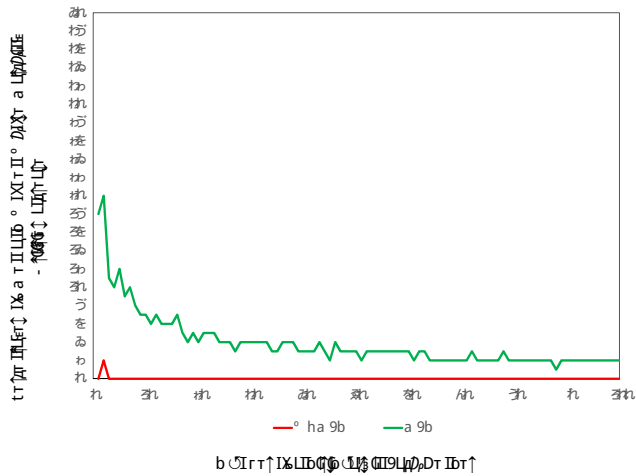
Figure 2. Percentages of Men and Women Who Are Made Worse Off by the Outcome of the DA-MP Algorithm After the Arrival of a Man



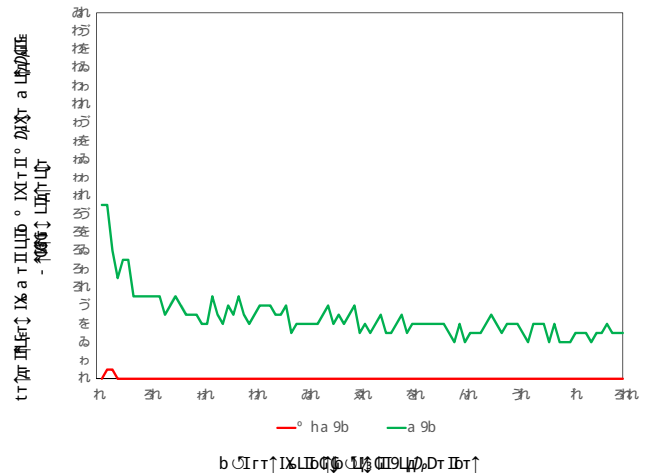
(a) $w = 0$



(b) $w = 1/3$

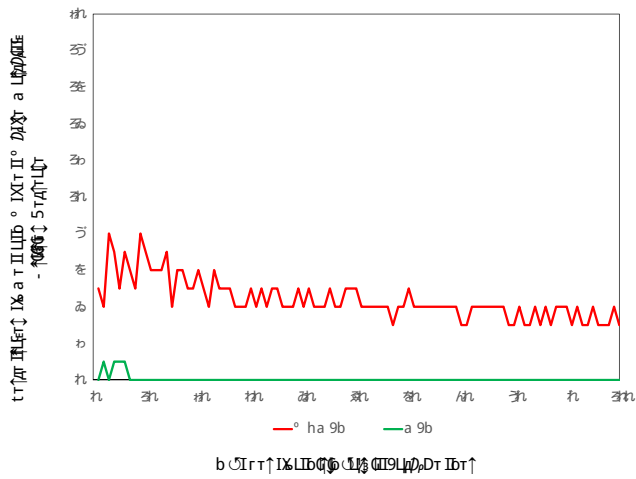


(c) $w = 2/3$

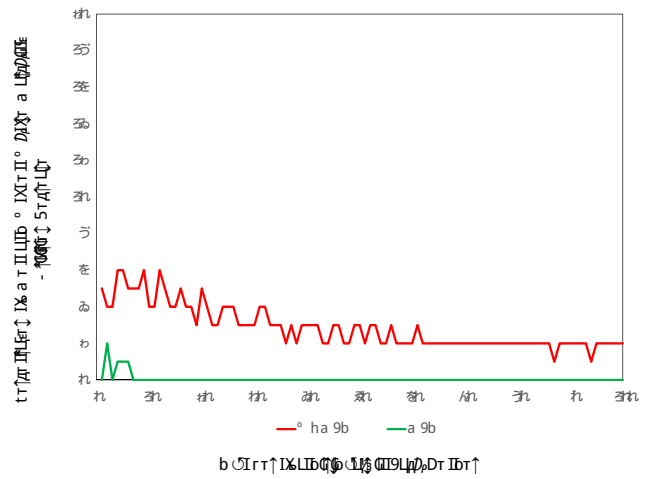


(d) $w = 1$

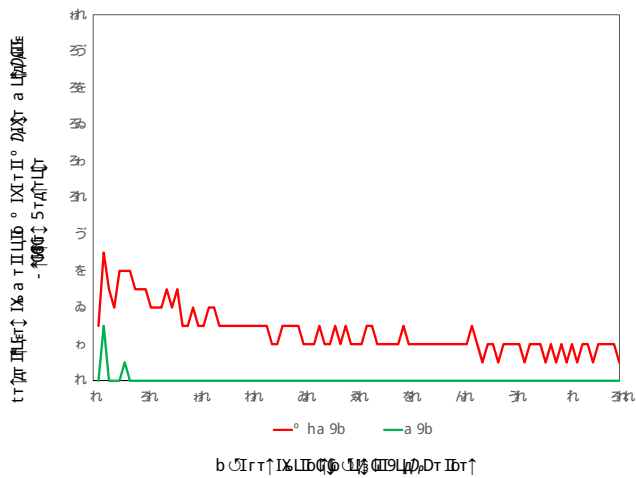
Figure 3. Percentages of Men and Women Who Are Made Better Off by the Outcome of the DA-MP Algorithm After the Arrival of a Woman



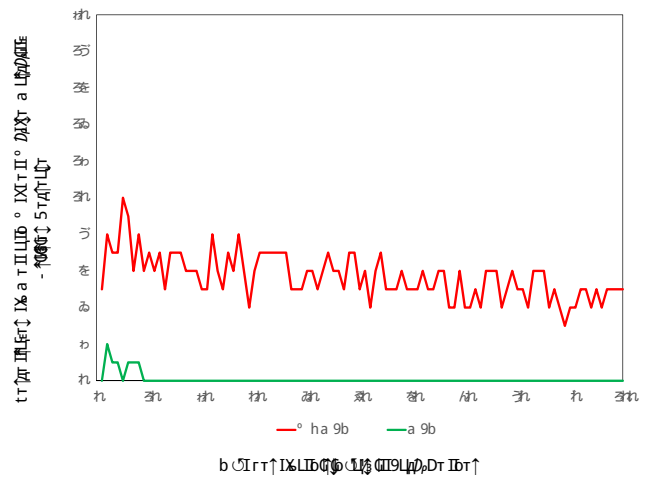
(a) $w = 0$



(b) $w = 1/3$

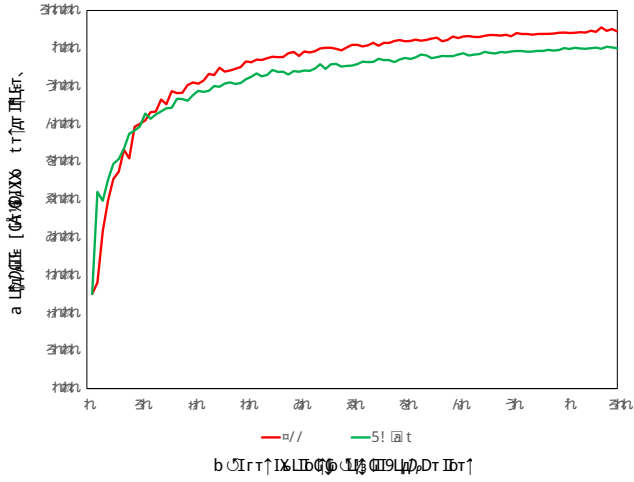


(c) $w = 2/3$

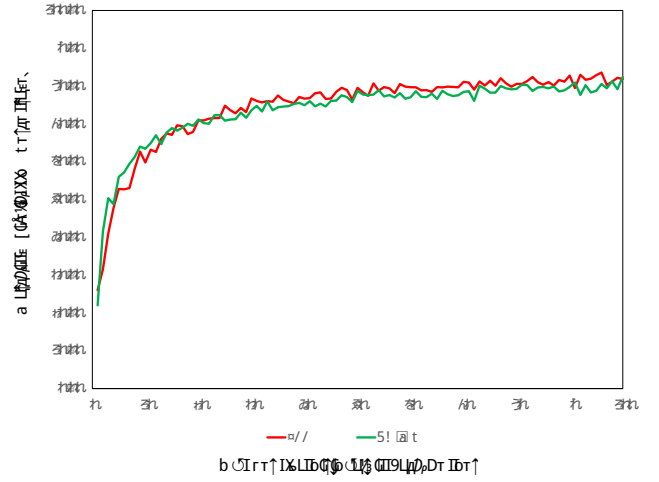


(d) $w = 1$

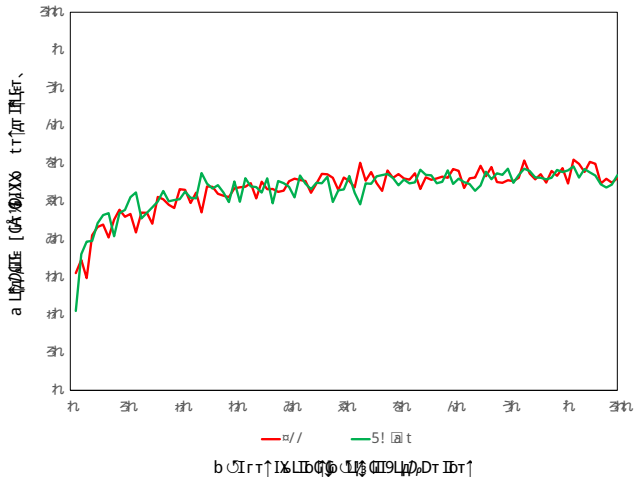
Figure 4. Percentages of Men and Women Who Are Made Worse Off by the Outcome of the DA-MP Algorithm After the Arrival of a Woman



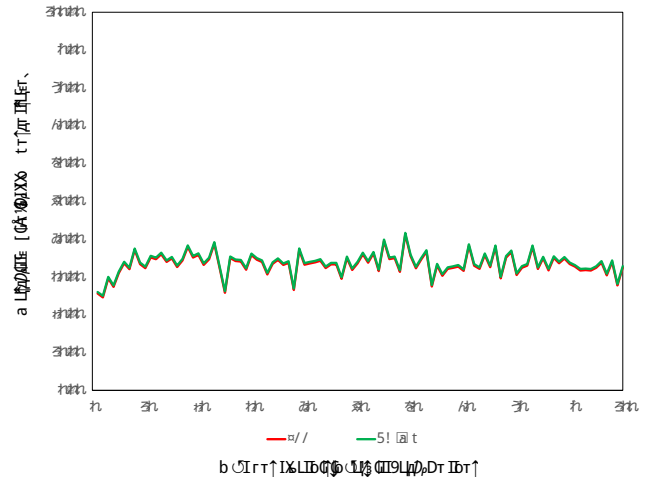
(a) $w = 0$



(b) $w = 1/3$



(c) $w = 2/3$



(d) $w = 1$

Figure 5. Likelihood of Matching Under the TTC-M and DA-MP Algorithms